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Continuing, $$A_1 = \frac{1}{(p-1)!}\frac{d^{p-1}}{dz^{p-1}}(z-\alpha)^p G(z)|_{z=\alpha}$$

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Through further differentiation,

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Through further differentiation, can derive Z-transforms of n^2, n^3, etc.

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$$\begin{aligned} u_a[t + T_p] &= A\cos(2\pi F(t + \frac{1}{F}) + \theta) \\ &= A\cos(2\pi + 2\pi F t + \theta) \\ &= A\cos(2\pi F t + \theta) \end{aligned}$$

9. Properties of Cont. Time Sinusoidal Signals

$$u_a(t) = A\cos(\Omega t + \theta)$$
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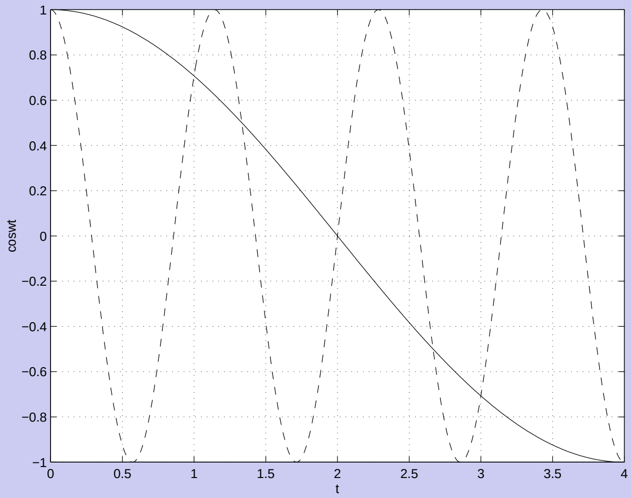

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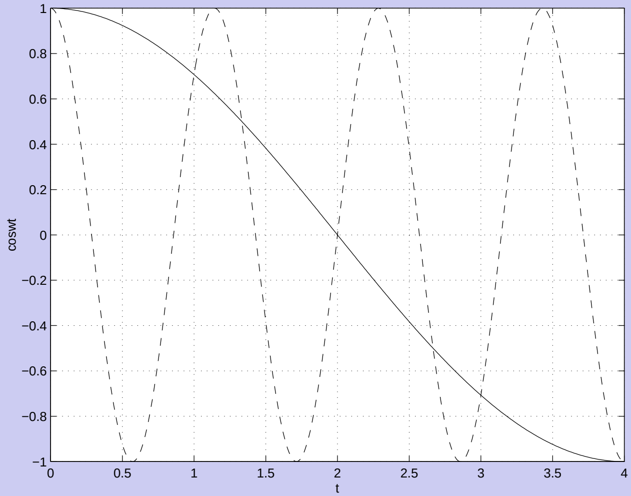

Different wave forms for different frequencies.

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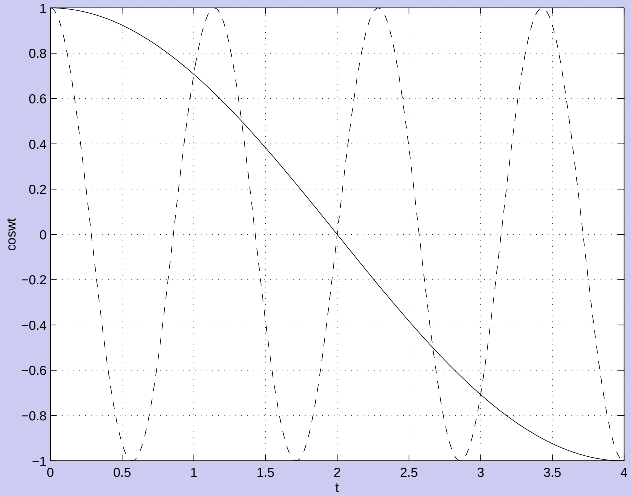

Different wave forms for different frequencies.

Can increase f all the way to ∞ or decrease to 0.

11. Discrete Time Sinusoidal Signals

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- f normalized frequency, cycles/sample

12. Discrete Time Sinusoids - Identical Signals

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Discrete time sinusoids whose frequencies are separated by integer multiple of 2π are identical, *i.e.*,

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$$-\pi < w_0 < \pi \text{ or } -\frac{1}{2} < f_0 < \frac{1}{2}$$

13. Sampling a Cont. Time Signal - Preliminaries

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$$-\frac{1}{2} < f < \frac{1}{2} \qquad -\pi < w < \pi$$

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w_2 is an alias of w_1

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17. Fourier Transform of Aperiodic Signals

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$$X(F) = \int_{-\infty}^{\infty} x(t)e^{-j2\pi Ft}dt$$

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If we let radian frequency $\Omega = 2\pi F$

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$x(t)$, $X[\Omega]$ are Fourier Transform Pair

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Apply $u(n) = e^{jwn}$ to $g(n)$ and obtain output y:

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$$= \sum_{k=-\infty}^{\infty} g(k)e^{jw(n-k)} = e^{jwn} \sum_{k=-\infty}^{\infty} g(k)e^{-jwk}$$

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Define Discrete Time Fourier Transform

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Define Discrete Time Fourier Transform

$$G(e^{jw}) \triangleq \sum_{k=-\infty}^{\infty} g(k)e^{-jwk}$$

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Define Discrete Time Fourier Transform

$$G(e^{jw}) \triangleq \sum_{k=-\infty}^{\infty} g(k)e^{-jwk} = \sum_{k=-\infty}^{\infty} g(k)z^{-k}\bigg|_{z=e^{jw}}$$

18. Frequency Response

Apply $u(n) = e^{jwn}$ to $g(n)$ and obtain output y:

$$y(n) = g(n) * u(n) = \sum_{k=-\infty}^{\infty} g(k)u(n-k)$$

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Provided the sequence converges absolutely

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- The system with large gains at low frequencies and small gains at high frequencies are called low pass filters
- Similarly high pass filters

20. Discrete Fourier Transform

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Define Discrete Time Fourier Transform

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Provided, absolute convergence

$$\sum_{k=-\infty}^{\infty} |g(k)e^{-jwk}| < \infty$$

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For causal systems, BIBO stability. Required for DTFT.

21. FT of Discrete Time Aperiodic Signals - Definition

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$$= \int_{-1/2}^{1/2} U(f)e^{j2\pi fm}df$$

22. FT of a Moving Average Filter - Example

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$$y(n) = \frac{1}{3}[u(n+1) + u(n) + u(n-1)]$$

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\begin{aligned}
y(n) &= \frac{1}{3}[u(n+1) + u(n) + u(n-1)] \\
y(n) &= \sum_{k=-\infty}^{\infty} g(k)u(n-k) \\
&= g(-1)u(n+1) + g(0)u(n) + g(1)u(n-1) \\
g(-1) &= g(0) = g(1) = \frac{1}{3} \\
G\left(e^{jw}\right) &= \sum_{n=-\infty}^{\infty} g(n)z^{-n}|_{z=e^{jw}} \\
&= \frac{1}{3}\left(e^{jw} + 1 + e^{-jw}\right) = \frac{1}{3}(1 + 2\cos w)
\end{aligned}
$$

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$$Arg(G) = \begin{cases} 0 & 0 \leq w < \frac{2\pi}{3} \\ \pi & \frac{2\pi}{3} \leq w < \pi \end{cases}$$

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```
// Updated (18-7-07)
// 5.3

w = 0:0.01:%pi;
subplot(2,1,1);
plot2d1("gll",w,abs(1+2*cos(w))/3,style = 2);
label('',4,' ','Magnitude',4);
subplot(2,1,2);
plot2d1("gln",w,phasemag(1+2*cos(w)),style = 2,rect =[0.
label('',4,'w','Phase',4)
```

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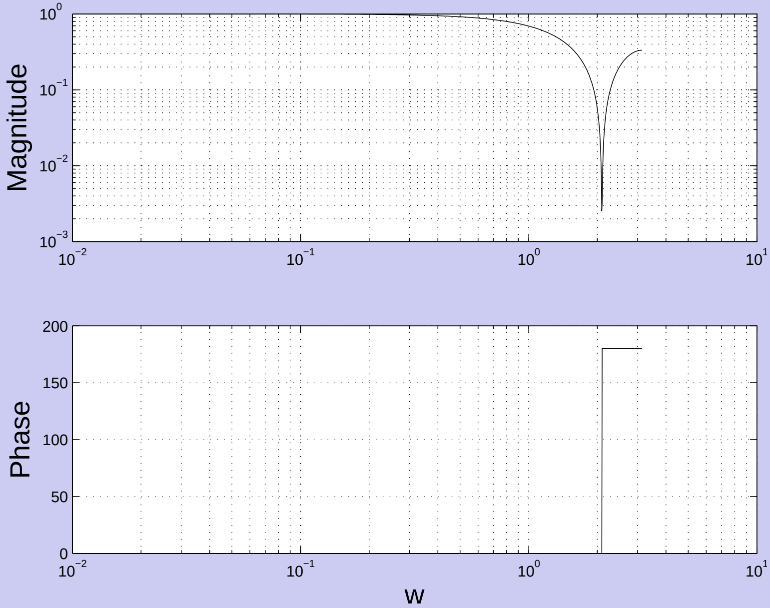

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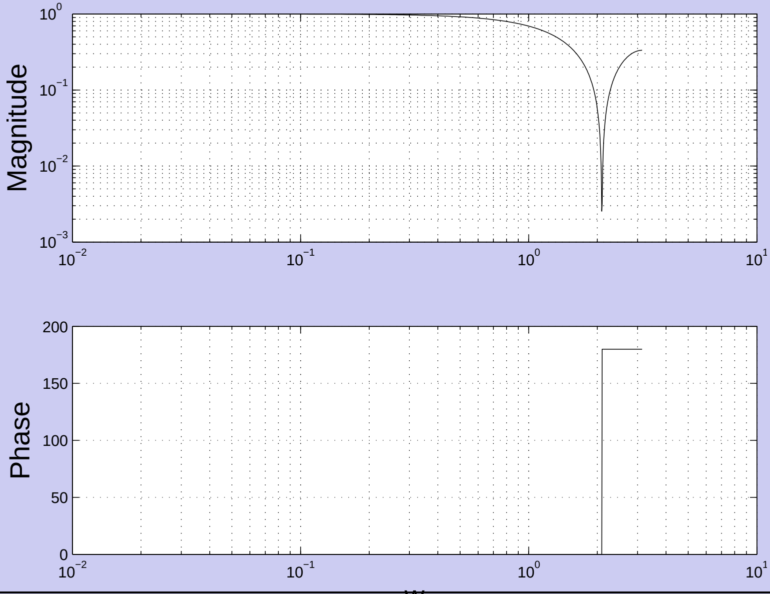

25. Additional Properties of Fourier Transform

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Symmetry of real and imaginary parts for real valued sequences

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$$= \sum_{n=-\infty}^{\infty} g(n)\cos wn - j\sum_{n=-\infty}^{\infty} g(n)\sin wn$$

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$$Re\left[G\left(e^{jw}\right)\right] = Re\left[G\left(e^{-jw}\right)\right]$$

$$Im\left[G\left(e^{jw}\right)\right] = -Im\left[G\left(e^{-jw}\right)\right]$$

26. Additional Properties of Fourier Transform

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Recall

$$\boldsymbol{G}\left(\boldsymbol{e}^{\boldsymbol{jw}}\right) = \boldsymbol{G}^{*}\left(\boldsymbol{e}^{-\boldsymbol{jw}}\right)$$

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Symmetry of magnitude for real valued sequences

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This shows that the magnitude is an even function.

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$\Rightarrow$ Bode plots have to be drawn for w in $[0, \pi]$ only.

27. Sampling and Reconstruction

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$$u(n) = u_a(nT_s), \quad -\infty < n < \infty$$

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Fast sampling:

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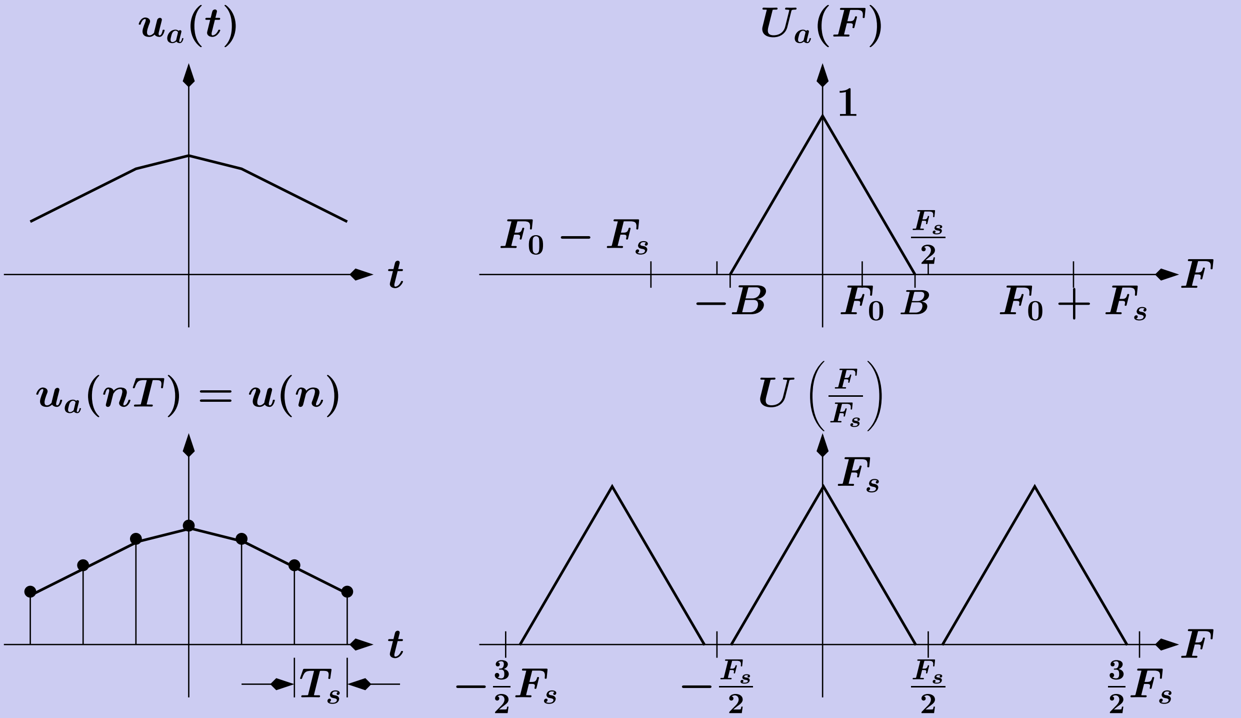

28. Slow Sampling Results in Aliasing

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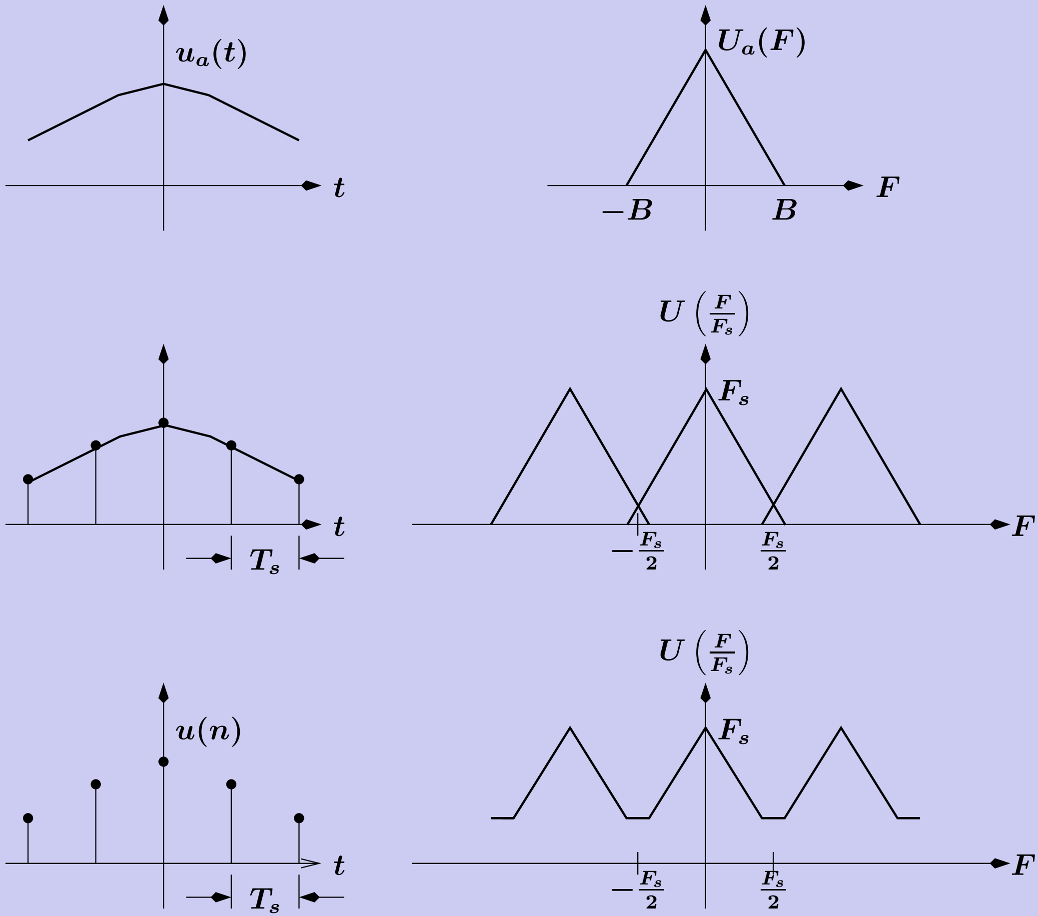

29. What to Do When Aliasing Cannot be Avoided?

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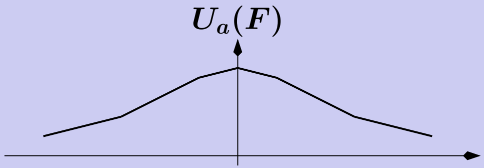
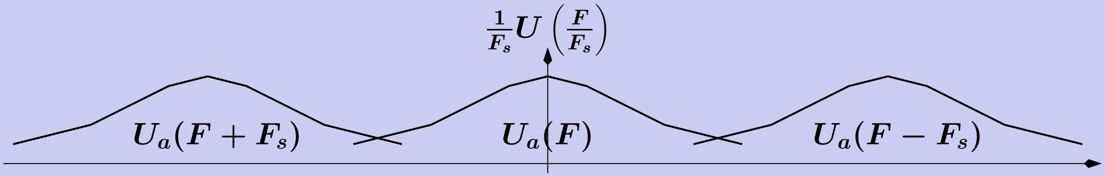
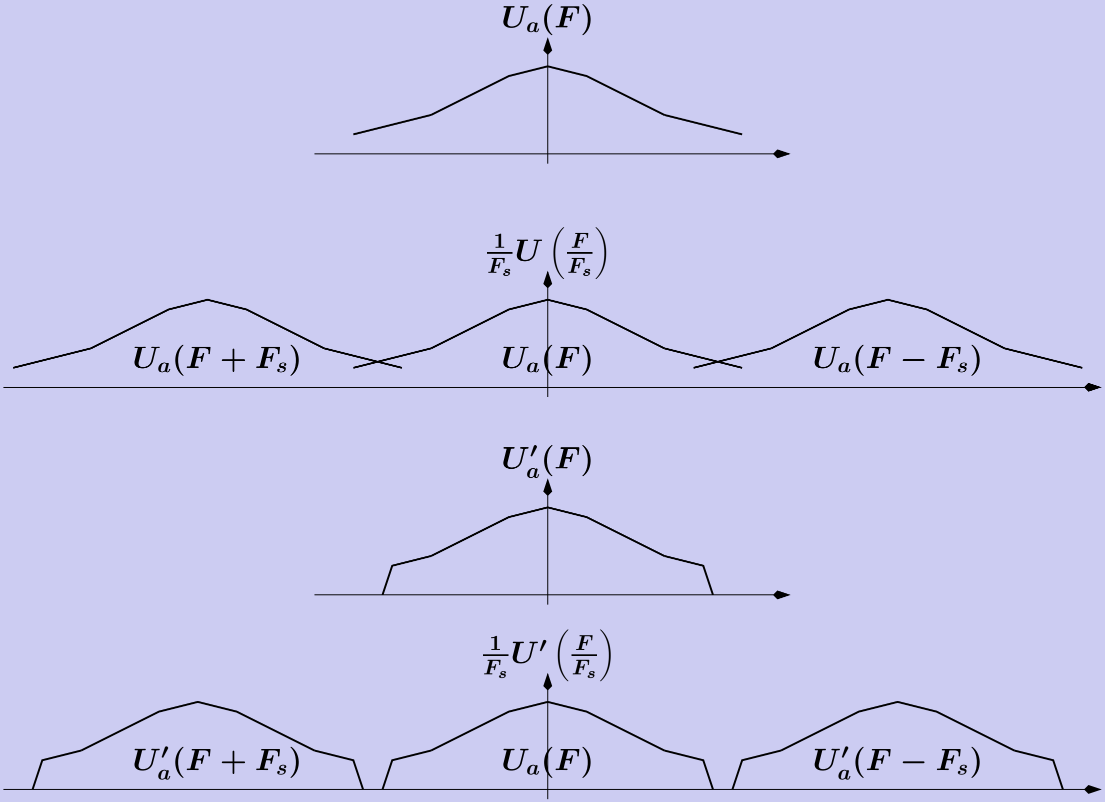




29. What to Do When Aliasing Cannot be Avoided?



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- Not causal: check $n > 0$

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  - $\boldsymbol{\omega_c T_s = 0.15}$ to 0.5, where, $\boldsymbol{\omega_c =}$ crossover frequency