1. Topics to be covered

• Introduction to modelling and identification

• Transfer function based control system design
  – PID control
  – Pole placement control
  – Minimum variance control
  – Model predictive control
  – Linear Quadratic Gaussian control
2. **Topics to be covered - continued**

- **State space control techniques**
  - Pole placement, controllability, observability
  - Linear quadratic regulator
  - Kalman filter
3. Digital Signals

- Digital signals: quantized in value, discrete in time
- Binary numbers 0, 1 used
- As 0 or 1 refers to a range of voltages, digital signals can be made less noisy
  - If transmitted signal is received exactly, no noise
  - Analog circuitry always has noise
  - Digital devices have good noise margins
4. TTL Noise Margin at Low State

Considered low if $< 0.4V$

Considered low if $< 0.8V$

Guaranteed noise margin $= 0.4V$ at low state!
5. TTL Noise Margin at High State

Considered high if \( > 2.4\text{V} \)

Considered high if \( > 2\text{V} \)

Guaranteed noise margin \( = 0.4\text{V} \) at high state too!

- Easy to implement/modify circuits - simply change the coefficients!
- Margins can handle noise, drift, etc.
- Can improve accuracy through more bits
- Can implement error checking protocols
- Can be reproduced in volumes
- Can be fully integrated through VLSI
- Through multiplexer, a single processor can handle a large number of digital signals

- So digital devices became popular - impetus for advancement of digital systems
- Digital devices have become rugged, compact, flexible and inexpensive
- Modern devices (controllers, filters, watches, computers, etc.) are digital
8. Analog to Digital (A/D) Conversion

A/D converter:

- samples analog signal
- produces binary equivalent (batch process, requires time)
- digital signal is quantized in value and discrete in time

\[\text{data} \xrightarrow{\text{A/D Converter}} \text{quantized data}\]

• Quantization errors
  – Finiteness of bits - quantization errors
  – Increase number of bits to reduce errors
  – Falling hardware prices help achieve this

• Sampling rate
  – Slow rate \(\Rightarrow\) loss of information
  – Fast rate \(\Rightarrow\) computational load

• Analog’s output is sent to digital through A/D Reverse?
10. Digital to Analog (D/A) Conversion

- Sampled signal

Real life systems are analog

Cannot work with binary numbers

Need to know values at all times
11. Digital to Analog (D/A) Conversion

- The easiest way to handle this is to use Zero Order Hold (ZOH)

- ZOH is the most popular

- We will consider only ZOH in this course
12. Digital to Analog (D/A) Conversion

- Assumption used in this course:
  - All inputs are ZOH signals
  - OK when the input is produced by a digital device
  - Also OK when the input signal varies slowly
13. Magnetically Suspended Ball

- Current through coil induces magnetic force
- Magnetic force balances gravity
- Ball is suspended in midair - 1 cm from core
- Want to move to another equilibrium
14. Magnetically Suspended Ball - Ctd.

**Force balance:**

\[
M \frac{d^2 h}{dt^2} = Mg - \frac{Ki^2}{h}
\]

**Voltage balance**

\[
V = L \frac{di}{dt} + iR
\]
15. Magnetically Suspended Ball - Ctd.

Model equations:

\[ M \frac{d^2 h}{dt^2} = Mg - \frac{Ki^2}{h} \]

\[ V = L \frac{di}{dt} + iR \]

In deviation variables:

\[ 0 = M \frac{d^2 h_s}{dt^2} = Mg - \frac{Ki^2_s}{h_s} \]

\[ M \frac{d^2 \Delta h}{dt^2} = - K \left[ \frac{i^2}{h} - \frac{i_s^2}{h_s} \right] \]
Linearize RHS:

\[
\frac{i^2}{h} = \frac{i_s^2}{h_s} + 2 \frac{i}{h} \bigg|_{(i_s, h_s)} \Delta i - \frac{i^2}{h^2} \bigg|_{(i_s, h_s)} \Delta h
\]

\[
= \frac{i_s^2}{h_s} + 2 \frac{i_s}{h_s} \Delta i - \frac{i_s^2}{h_s^2} \Delta h
\]
17. Magnetically Suspended Ball - Ctd.

\[
\frac{i^2}{h} = \frac{i_s^2}{h_s} + 2\frac{i_s}{h_s} \Delta i - \frac{i_s^2}{h_s^2} \Delta h
\]

\[
M \frac{d^2 \Delta h}{dt^2} = -K \left[ \frac{i^2}{h} - \frac{i_s^2}{h_s} \right]
\]

Substitute and simplify

\[
M \frac{d^2 \Delta h}{dt^2} = -K \left[ \frac{i_s^2}{h_s} + 2\frac{i_s}{h_s} \Delta i - \frac{i_s^2}{h_s^2} \Delta h - \frac{i_s^2}{h_s} \right]
\]

\[
\frac{d^2 \Delta h}{dt^2} = \frac{K i_s^2}{M h_s^2} \Delta h - 2\frac{K i_s}{M h_s} \Delta i.
\]
Force and Voltage balance:

\[
\frac{d^2 \Delta h}{dt^2} = \frac{K}{M h_s^2} \Delta h - 2\frac{K i_s}{M h_s} \Delta i
\]

\[
\Delta V = L \frac{d\Delta i}{dt} + R \Delta i
\]

Define new variables and hence, equations:

\[
x_1 \triangleq \Delta h, \quad x_2 \triangleq \Delta \dot{h}, \quad x_3 \triangleq \Delta i, \quad u \triangleq \Delta V
\]

\[
\frac{dx_1}{dt} = x_2, \quad \frac{dx_2}{dt} = \frac{K}{M h_s^2} x_1 - 2\frac{K i_s}{M h_s} x_3
\]

\[
\frac{dx_3}{dt} = -\frac{R}{L} x_3 + \frac{1}{L} u
\]
19. Magnetically Suspended Ball - Ctd.

\[
\begin{align*}
\frac{dx_1}{dt} &= x_2, \quad \frac{dx_2}{dt} = \frac{K}{M h_s^2} i_s^2 - 2\frac{K}{M h_s} x_1 - 2\frac{K}{M h_s} x_3 \\
\frac{dx_3}{dt} &= -\frac{R}{L} x_3 + \frac{1}{L} u
\end{align*}
\]

In matrix form:

\[
\frac{d}{dt} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ \frac{K}{M h_s^2} i_s^2 & 0 & -2\frac{K}{M h_s} \\ 0 & 0 & \frac{R}{L} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1/L \end{bmatrix} u
\]

\[
\dot{x}(t) \triangleq Fx(t) + Gu(t)
\]
20. Model of a Flow System

\[ A \frac{dh(t)}{dt} = Q_i(t) - Q_o(t) \]

\[ Q_o(t) = k \sqrt{h(t)} \]

\[ A \frac{dh(t)}{dt} = Q_i(t) - k \sqrt{h(t)} \]

\[ \Delta h(t) \triangleq h(t) - h_s, \quad \Delta Q_i(t) \triangleq Q_i(t) - Q_{is} \]

\[ A \frac{d\Delta h(t)}{dt} = \Delta Q_i(t) - \frac{k}{2\sqrt{h_s}} \Delta h(t), \quad (IC) \]

\[ \dot{x} = Fx + Gu \]
21. DC Motor

- DC Motor: popular rotary actuator
- On application of an electrical voltage, the rotor rotates, as per Newton’s law of motion:

\[
\frac{J}{b} \ddot{\theta} = -\dot{\theta} + \frac{K}{b} V
\]

where, \(\theta\) and \(J\) are the angular position and the moment of inertia of the shaft, respectively.
- \(V, b, K\) are voltage, damping factor, a constant
Recall the model: \( \frac{J}{b} \ddot{\theta} = -\dot{\theta} + \frac{K}{b} V \)

- The initial angular position and the angular velocity may be taken to be zero. Suppose \( K/b = 1 \).
- If we define \( x_1 = \dot{\theta} \), \( x_2 = \theta \), we obtain \( \dot{x} = Fx + Gu \) with

\[
\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \quad F = \begin{bmatrix} -b/J & 0 \\ 1 & 0 \end{bmatrix}, \quad G = \begin{bmatrix} b/J \\ 0 \end{bmatrix}.
\]

\( x_1(0) = 0 \) and \( x_2(0) = 0 \).

- Same model can be used for satellite tracking antenna systems and ships, if \( J/b \) is interpreted as the time constant.
23. IBM Lotus Domino Email Server

- Clients access the database of emails maintained by the server through Remote Procedure Calls (RPCs).
- Number of RPCs, denoted as RIS has to be controlled.
- If the number of RIS becomes large, the server could be overloaded, with a consequent degradation of performance.
- If RIS is less, the server is not being used optimally.
- Not possible to regulate RIS directly.
• Regulation of RIS may be achieved by limiting the maximum number of users ($\text{MaxUsers}$) who can simultaneously use the system.

• Because of stochastic nature, difficult to come up with analytic model.

• Obtained through expt., data collection, curve fitting (identification).

\[
y(k) = \text{RIS}(k) - \overline{\text{RIS}} \\
u(k) = \text{MaxUsers}(k) - \overline{\text{MaxUsers}} \\
y(k + 1) = 0.43y(k) + 0.47u(k)
\]
Mix of Analog and Digital Devices: Control

- \( e \) is converted to digital signal using A/D converter
- \( u \) is made useful to real life system with D/A converter and ZOH
- If the plant is naturally discrete time, \( x(n + 1) = Ax(n) + Bu(n), y(n) = Cx(n) + Du(n) \), no problem
- What if it is continuous time?
26. Motivation for Discrete Model

• Real systems are continuous

• Digital system’s view of the process:
  – Receives and sends out sampled signals only
  – Thus views the process as a sampled system

• Can understand only discrete time models
  – Discrete model relates the system variables as a function of their values at previous time instants
  – No value required/used in between sampling instants. Time derivatives have no meaning
27. State Space Models

- Model is of the form \( \dot{x} = Fx(t) + Gu(t) \)
- \( x \) state
  - denotes variables that characterize the state
  - knowing state, know *everything* about system
- \( u(t) \) denotes the input to the system:
  - disturbance/manipulated/control variable
- In the flow system,
  - Inflow rate \( F_i \) is the disturbance variable
  - Valve position is manipulated/control variable
28. Textbook

Title: Digital Control
Author: Kannan Moudgalya
Publisher: John Wiley & Sons
Place: Chichester, UK
Year: 2007
ISBN: 978-0-470-03143-8
29. **Textbook: Think Digital**

- Teaches digital control from **scratch**
  - Analog control background is not necessary
  - Suitable for students whose domain consists of only discrete time systems, for example, computing systems and supply chain systems
  - Gives another perspective to students good in analog control: helps improve understanding
  - Suitable also for students who return to studies after a break
- **Helps think digital**
What are the Benefits?

- Filter design is easier than continuous time
- Controller implementation is easier
- Identification is easier
- Deadbeat control, minimum variance control, model predictive control, natural in discrete time domain
31. **Textbook: Contents**

- Detailed introduction to DSP - only discrete time
- Detailed introduction to identification
- The above two alone can make one course
- Transfer function approach to control design
- State space approach to control design
Control Design Techniques

• PID controller discretization
• Pole placement
• Special cases of pole placement
  – Smith predictor
  – Internal model control
• Minimum variance control
• Model predictive control
• Linear quadratic Gaussian control
33. Matlab Programs

- Control design techniques supported by matlab code
- Textbook is the manual for the programs
- Programs are listed in the book
- Index of matlab code
- Matlab code is available at www.moudgalya.org
- Scilab equivalent also available at the same website
  - Scilab programs have the same prefix as the matlab programs, but with extension sci or sce
  - Scicos programs have the same prefix as simulink code, but with cos as the extension
34. References
